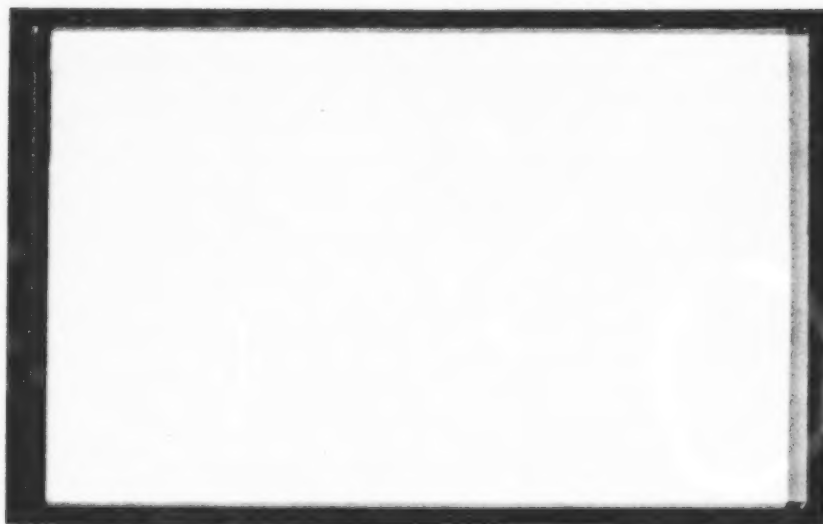


UNIVERSITÉ DE MONTRÉAL



CENTRE DE RECHERCHE SUR LES TRANSPORTS



CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

OBSERVATIONS ON THE STOCHASTIC
VEHICLE ROUTING PROBLEM

by

Moshe Dror* and Pierre Trudeau**

* Industrial Engineering and Management,
Ben-Gurion University of the Negev.
Visiting Professor, Centre de recherche sur les transports,
Université de Montréal, Summer 1984.

** Centre de recherche sur les transports,
Université de Montréal

Université de Montréal
Centre de recherche sur les transports - Publication #368
Septembre 1984

ABSTRACT

A stochastic vehicle routing problem (SVRP) differs from the vehicle routing problem (VRP) in that the demand is probabilistic. One aspect that differentiates between these problems is the notion of route failure. A route failure occurs only in the SVRP, when the real demand on a route exceeds the vehicles capacity. In this paper, we illustrate the effects of route failure on the expected cost of a route, as well as the impact the direction of a designed route can have on the expected cost. We also present an algorithm which more fully accounts for all the costs inherent in the SVRP.

1. PROBLEM DESCRIPTION

A vehicle routing problem (VRP) is the problem of designing a minimum cost set of routes for a fleet of delivery (pickup) vehicles of fixed capacity. The routes must be designed so that

- a) each demand location is served by exactly one vehicle,
 - b) the total demand on each route is less than or equal to the capacity of the vehicle assigned to that route,
 - c) each route begins and ends at the central depot,
- and in many instances there is an additional requirement that,
- d) the time required to complete a route is less or equal to some prespecified constant (usually related to the length of driver's workday).

In the VRP, it is assumed that the demand at each location is known before the routes are designed. The stochastic VRP differs from the VRP in that the demand at each location is a random variable whose value is unknown until the vehicle arrives at that location. According to Stewart and Golden [9], the objective when solving the stochastic VRP is to design a set of routes that minimizes the expected daily costs of the routing system. The expected daily cost of the routing system consists of the expected cost of the designed routes, plus the expected cost of servicing those customers who could not be serviced by the designed routes. Since the actual demand on some routes may exceed the capacity of the vehicles assigned to them, all customers will not necessarily be serviceable by the designed routes alone. We introduce the term "route failure" for the case in which the total actual demand on a route exceeds the capacity of the vehicle assigned to that route.

A single vehicle route consists of : the set of customers, the order of their assigned service, their demand characteristics and location, and the vehicle's capacity.

In this paper we examine the expected service (routing) cost for the route's customers and the probability of route failure.

Review of the literature

The VRP has been treated by many researchers. Bodin et al. [1] provide an overview and describe the state of the art in vehicle routing, together with a list of over 700 references. In contrast, the stochastic vehicle routing problem has received far less attention; this is reflected in the operations research literature. To our knowledge the most recent work in this area is that of Stewart and Golden [9] and Stewart et al. [10] which extends and generalizes the earlier results. The latter paper attempts to pursue the results as presented in Stewart and Golden [9]. Rather than referring the reader back to [9] repeatedly as we analyze the stochastic VRP models presented there, we restate their models below :

Chance-constrained model (CCM)

$$\begin{aligned} \text{Minimize} \quad & \sum_k \sum_{i,j} c_{ij} x_{ijk} \\ \text{Subject to} \quad & \Pr \left\{ \sum_{i,j} d_i x_{ijk} < Q \right\} > 1-\alpha, k=1, \dots, m \\ & x = [x_{ijk}] \in S_m \end{aligned}$$

where

c_{ij} = the cost of traveling from i to j

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

m = the number of vehicles available

S_m = the set of all feasible solutions to the m -traveling salesman problem

d_i = random variable representing the demand at location i

α = the maximum allowable probability that a route fails.

Q = the vehicle capacity.

This model belongs to class of models referred to as chance-constrained programming developed by Charnes and Cooper (see [2,3]).

Two other models presented in [9] belong to the class of stochastic models with recourse in which when a constraint fails there is a penalty associated with regaining feasibility. In the first model, the penalty associated with a constraint violation is independent of the degree of violation; this corresponds to the number of units by which demand exceeds vehicle capacity on that route. In the second model, the penalty is proportional to the degree of constraint violation.

Penalty Model 1 (PM1)

$$\begin{aligned} &\text{Minimize} \quad \sum_k \sum_{i,j} c_{ij} x_{ijk} + \sum_k \lambda_k \Pr\left\{\sum_{i,j} d_i x_{ijk} > Q\right\} \\ &\text{subject to} \quad x \in S_m \end{aligned}$$

λ_k = fixed penalty incurred each time route k fails.

Penalty Model 2 (PM2)

$$\begin{aligned} &\text{Minimize} \quad \sum_k \sum_{i,j} c_{ij} x_{ijk} + \sum_k \varepsilon_k E[\lambda_k] \\ &\text{Subject to} \quad x \in S_m \end{aligned}$$

ε_k = the penalty per unit of demand in excess of Q on route k .

$E[\lambda_k]$ = the expected number of units by which demand will exceed capacity on route k .

$$E[\lambda_k] = \int_0^{\infty} \lambda_k f\left(\sum_{i,j} d_i x_{ijk} - Q = \lambda_k\right) d\lambda_k$$

f = the probability density function of λ_k - the number of units in excess of capacity on route k .

Our first observation is in regard to the two penalty models. Given a route, say the k^{th} route, the x_{ijk} variables for this route are fixed. This immediately fixes the corresponding c_{ij} 's and the first term of the objective function, i.e., $\sum_k \sum_{i,j} c_{ij} x_{ijk}$ gets its value. The second term expresses the expected cost for the route violations. Implicit in the objective is that we visit all the customers assigned to the route regardless of the inventory position of the vehicle. But if for instance, the vehicle's inventory got completely depleted at the second to last customer on the route, regardless of the last customer's demand, the vehicle will return directly to the depot. Thus, the c_{ij} costs associated with getting to the last customer, and from there to the depot will not be realized; instead the route will incur the cost of returning directly to the depot on the depletion of the inventory on the vehicle. In short, the objective function in both penalty models requires modification. In subsequent sections of this paper we will attempt to understand the routing cost behaviour of the stochastic VRP.

2. PROBABILITY PROPERTIES OF ROUTE FAILURE

Consider a single route. The corresponding VRP is simply the classical traveling salesman problem (TSP). Given n customers, and a depot, the possible number of different TSP routes is $n!$. Let R denote the set of all the traveling salesman routes and for $r \in R$ (r denotes a particular route) let r_0 denote the depot, r_1 the first customer on the route r , and in general, r_i the i^{th} customer, r_n the last customer on route r before returning to the depot. The routing cost for route r is given by

$\sum_{i=1}^n c(r_{i-1}, r_i) + c(r_n, r_0)$. Denote by $P(r_i)$ the probability of route failure at the i^{th} location on route r .

Lemma 1 : Given that the customers' demands are independent random variables with nonnegative means, then for any $r \in R$, $\{P(r_i)\}_{i=1}^n$ is a nondecreasing sequence.

In fact, the function $P(r_i)$ for any $r \in R$ is a nondecreasing step function in i . This property of $P(r_i)$ is independent of the route choice between the $n+1$ locations (including the depot). Consequently, from now on, we can drop r in this notation and refer to $P(i)$ as the probability of route failure at the i^{th} location on a route.

Definition ([5]) : Given a finite or infinite sequence x_i , $i \in I$, we say that the sequence $\{x_i\}$, $i \in I$ is convex if for all $i \in I$, $x_{i-1} - x_i > x_i - x_{i+1}$.

The sequence $\{x_i\}$, $i \in I$ is strictly convex if $x_{i-1} - x_i > x_i - x_{i+1}$ for all $i \in I$.

Lemma 2 : Given that the customers' demands on a route (i.e., the d_i 's) are independent, identical, normally distributed random variables with $\sigma < \mu > 0$, and that $E[\sum_{i=1}^n d_i] + \epsilon < Q$ for some $\epsilon > 0$, the sequence $\{P(i)\}_{i=1}^n$ is convex. If $\mu > 0$ then the sequence $\{P(i)\}_{i=1}^n$ is strictly convex. Proof is given in [5, p.28] and [6].

Conjecture : The normality assumption in Lemma 2 can be dropped and the results will still hold (i.e., the results are essentially independent of the demands' distribution, they depend only on the fact that the individual demands are independent and identically distributed random variables with $0 < \mu$ and $\sigma < \mu$).

Note : The properties of the sequence $\{P(i)\}_{i=1}^n$ are independent of the vehicle's capacity, regardless of whether it represents the actual capacity Q , or some artificial capacity $\bar{Q}$, as in the transformation from (CCM) to VRP. (See [9, p. 378]).

3. LOCAL PROBABILITY OF ROUTE FAILURE AND ROUTE DIRECTION

Denote by $\mu_i = E[d_i]$ and $\sigma_i^2 = \text{Var}[d_i]$. Given that $\sum_{i=1}^n \mu_i < Q$, can the probability of route failure at a specific location be quantified? The answer is yes under some proper assumptions about the distributions of d_i 's. As we pointed out, the models presented by Stewart and Golden in [9] implicitly assume visits to all customers' locations according to the designed route structure. This assumption can be defended only in the case when the probability of route failure up to the $(n-1)^{\text{st}}$ location is negligible in comparison to that at the n^{th} location. Stated differently : the ratio $P(n)/P(n-1)$ is greater than some constant, greater than say 10. We examine the assumptions under which this is true. Note that one interpretation of such a result would be that if a route fails, it fails in great majority of the cases at the last customer's location.

With a simple example we demonstrate that generally the probability of route failure at or before the $(n-1)^{\text{st}}$ customer cannot be disregarded when compared with the probability of route failure at the n^{th} - last customer on the route. We know from Lemma 1 that $\{P(i)\}_{i=1}^n$ is a nondecreasing sequence. How should this fact of nondecreasing probabilities of route failure impact on the design of a vehicle route? Who should be set as the last customer on a route?

Another question : Given a route on an undirected network, is the expected cost of servicing this route the same regardless of whether the planned order of the customers' visits is reversed? Again, a simple example demonstrates how critically the order of customer servicing can affect the route's expected cost.

Example 1 : Calculation of the probability of route failure at or before the i^{th} customer on a route.

Assume a route consisting of 20 customers. We consider two cases : one in which the customers' demand has the form of a uniform distribution and one in which the demand is normally distributed.

Case (i) : Customers' demands are independent, identically distributed, uniform random variables $d_i (i=1, \dots, 20)$ where $d_i \in (0, \frac{Q}{10}]$ and Q is the vehicle's capacity.

For each customer the mean demand equals $\frac{Q}{20}$ and the variance equals $\frac{Q^2}{1200}$. We examine the probability that the demand of the first n customers exceeds the vehicle's capacity.

$$P(n) = \Pr\left\{\sum_{i=1}^n d_i > Q\right\}$$

Note that for $n < 10$ $P(n) = 0$. For $n > 10$ we make use of the central limit theorem and say that

$$\sum_{i=1}^n d_i \approx N\left(n \frac{Q}{20}, n \frac{Q^2}{1200}\right)$$

(i.e., the sum is approximately normally distributed with the appropriate mean and variance).

$$\begin{aligned} P(n) &= \Pr\left\{\sum_{i=1}^n d_i > Q\right\} \approx \Pr\left\{Z > \frac{Q - \frac{nQ}{20}}{\sqrt{n \frac{Q^2}{1200}}}\right\} \\ &= \Pr\left\{Z > \frac{20-n}{\sqrt{n/3}} = t_n\right\}. \end{aligned}$$

Table 1

n	P(n)	ratio = $\frac{P(n)}{P(n-1)}$
11	.0000	----
12	.0000	----
13	.0004	----
14	.0027	6.75
15	.0125	4.63
16	.0418	3.34
17	.1038	2.48
18	.2061	1.99
19	.3450	1.67
20	.5000	1.45

Case (ii) : Customers' demands are independent, identical, normally distributed, random variables d_i . We examine the behaviour of $P(n)$ for two different values of the coefficient of variation $c = \sigma/\mu$.

a) $c = 1/2, d_i : N(\mu = \frac{Q}{20}, \sigma^2 = (\frac{Q}{40})^2)$

and b) $c = \frac{20}{21}, d_i : N(\mu = \frac{Q}{20}, \sigma^2 = (\frac{Q}{21})^2)$

Table 2

C = 1/2			C = 20/21	
n	P(n)	P(n)/P(n-1)	P(n)	P(n)/P(n-1)
9	.0000	----	.0001	----
10	.0000	----	.0005	5.00
11	.0000	----	.0022	4.40
12	.0000	----	.0078	3.55
13	.0001	----	.0207	2.65
14	.0007	7.00	.0465	2.25
15	.0049	7.00	.0869	1.87
16	.0228	4.65	.1469	1.69
17	.0721	3.16	.2236	1.52
18	.1736	2.41	.3085	1.38
19	.3228	1.86	.4052	1.31
20	.5000	1.55	.5000	1.23

Tables 1 and 2 illustrate that when examining the probability of route failure in the case of stochastic vehicle routing, one cannot discard completely the probabilities of route failure before reaching the last customer. As Table 1 indicates, the ratio $P(n)/P(n-1)$ for the case of uniformly distributed demand decreases to the value of 1.45. The probability, in our example, of never reaching the n^{th} customer is .345 and subsequently the cost $c_{n-1,n}$ of going from the $(n-1)^{\text{st}}$ customer to the n^{th} customer will not occur in 34.5% of the cases of servicing this route. Similar results can be concluded when the customers' demand is normally distributed as illustrated by Table 2.

Example 2 : Assume a route with five customers and a depot. The depot is located at (0,0) and the customers are located at (5,0), (5,5), (0,5), (-5,5), (-5,0); i.e., forming a rectangle of length 10 and width 5. They are denoted as customers 1, 2, 3, 4 and 5 respectively. Assume a straight line distance between the different locations and the following expected demand at the locations : .15Q for customers 1, 2, 3 and 4 and .40Q for customer 5 located at (-5,0). We also assume that in the event of a route failure each not fully serviced customer will receive an individual delivery. The cost is given in terms of the distance traveled. We examine the expected cost of a route starting at (0,0), and then (5,0) as the first customer and continuing to (5,5), (0,5), (-5,5), (-5,0) and back to the depot (0,0). This route is denoted as C^R . The route in the opposite direction : (0,0), (-5,0), (-5,5), (0,5), (5,5), (5,0), (0,0) is denoted as C^L .

$$\begin{aligned}
 \text{Expected cost of } C^R &= c_{01} + \Pr\{d_1 > Q\} \cdot [c_{10} + 2(c_{01} + c_{02} + c_{03} + c_{04} + c_{05})] \\
 &+ \Pr\{d_1 < Q \text{ and } d_1 + d_2 > Q\} \cdot [c_{12} + c_{20} + 2(c_{02} + c_{03} + c_{04} + c_{05})] \\
 &+ \Pr\{d_1 + d_2 < Q \text{ and } d_1 + d_2 + d_3 > Q\} \cdot [c_{12} + c_{23} + c_{30} + 2(c_{03} + c_{04} + c_{05})] \\
 &+ \Pr\{d_1 + d_2 + d_3 < Q \text{ and } d_1 + d_2 + d_3 + d_4 > Q\} \cdot [c_{12} + c_{23} + c_{34} + c_{40} + 2(c_{04} + c_{05})] \\
 &+ \Pr\{d_1 + d_2 + d_3 + d_4 < Q \text{ and } d_1 + d_2 + d_3 + d_4 + d_5 > Q\} \cdot [c_{12} + c_{23} + c_{34} + c_{45} + c_{50} + 2c_{05}] \\
 &+ \Pr\{d_1 + d_2 + d_3 + d_4 + d_5 < Q\} \cdot [c_{12} + c_{12} + c_{34} + c_{45} + c_{50}]
 \end{aligned}$$

The expected cost of C^L is computed in a similar manner.

Given that customers' demands are independent, identical, normally distributed random variables with mean demand as given and the same ratios of $\sigma/\mu = 0.5$ we obtain significantly different expected costs for servicing the customers if the route planned is (0, 1, 2, 3, 4, 5, 0) or is (0, 5, 4, 3, 2, 1, 0).

The expected cost of C^R equals 40.5563 and the expected cost of C^L equals 48,8362. The examples 1 and 2 illustrate that the mathematical models (CCM), (PM1) and (PM2) do not address all the complexities arising when customers' demand is considered to be stochastic.

4. THE ADAPTATION OF CLARK AND WRIGHT ALGORITHM

Stewart and Golden [9] apply the algorithms they developed for the stochastic VRP on several problems with stochastic demands. These problems are extensions of the 50-customer and 75-customer problems of Christofides and Eilon [4]. The problems were constructed by adding a demand standard deviation at each delivery location. The original demand quantity was used as the mean demand at each location. The standard deviations in demand were generated using a uniform random number generator so that they fall between zero and one-third of the mean demand at each delivery location to be independent of each other and normally distributed.

In this paper, exactly the same procedure was adopted on the same 75-customer problem from [4] to generate a stochastic vehicle routing problem. The objective in the case of the stochastic vehicle routing problem is to design vehicle routes which minimize the expected routing costs. The Clark and Wright algorithm is one of the most popular heuristic procedures for designing a feasible solution for a vehicle routing problem. We modify the Clark and Wright algorithm in order to design a set of routes for the SVRP with the goal of minimizing the expected routing costs. The thrust of the Clark and Wright algorithm is the idea that "savings" can be realized if two delivery nodes on two different routes can be joined by one route. In the VRP, the savings s_{ij} are calculated for each pair of delivery points (i,j) as $s_{ij} = c_{oi} + c_{oj} - c_{ij}$ for $i,j=1,\dots,n$ (n is the number of customers).

The algorithm orders these savings and proceeds to join nodes on the routes according to the largest saving available, as long as the demand of the combined route does not exceed the vehicle's capacity- Q . Both nodes i and j currently examined have at least one connection to the central depot. This basic savings idea is generalized for the stochastic vehicle routing case as follows :

s_{ij} = [the expected cost of the route with the customer i on it] + [the expected cost of the route with the customer j on it] - [the expected cost of the combined route where customer i immediately precedes customer j].

An other important difference between this algorithm and the Clark and Wright algorithm is that here the s_{ij} terms are recalculated each time after joining two routes. Other than that it is basically a Clark and Wright algorithm. We call this adaptation of Clark and Wright the Stochastic Clark and Wright algorithm (SC-W).

When computing the savings terms in the SC-W one has to be careful because the direction of the routes is an important factor and should be taken into account in the calculation. For each pair of nodes (i,j) two different directional situations have to be taken into account, and only the one with the highest value is considered (i.e., $\max \{s_{ij}, s_{ji}\}$).

In order to illustrate more fully the implications of the computational results, and to make their reproduction possible, we restate the data of the 75-customer problem; i.e., the customers' coordinates, their mean demand and their variance in Table 3.

The solution to the 75-customer stochastic vehicle routing designed by the SC-W algorithm is presented in Table 4. The number of routes in this solution is 10 and the probability that some of these routes will fail is considerable (.679 in the worst case - route 10). Note that the mean demand on route 10 exceeds the truck capacity by 6 units. This fact is responsible for the high probability of failure for this route. When we examined route 10 carefully, we noted that the last customer to join this route was customer 75 with a higher than average mean demand, and located very close to the depot. The direction of this route is such that this

Table 3 : Vehicle routing problem with probabilistic demands.
Central depot has coordinates (40, 40). Vehicle capacity is 160 units.

No.	x	y	d	σ^2	No.	x	y	d	σ^2	No.	x	y	d	σ^2	No.	x	y	d	σ^2
1	22	22	18	12.115	20	66	14	22	25.893	39	30	60	16	10.086	58	40	60	21	6.405
2	36	26	26	67.861	21	44	13	28	76.070	40	30	50	33	5.316	59	70	64	24	4.094
3	21	45	11	8.314	22	26	13	12	0.720	41	12	17	15	19.719	60	64	4	13	1.485
4	45	35	30	8.858	23	11	28	6	0.068	42	15	14	11	2.896	61	36	6	15	3.578
5	55	20	21	10.086	24	7	43	27	60.270	43	16	19	18	14.199	62	30	20	18	0.315
6	33	34	19	0.002	25	17	64	14	14.604	44	21	48	17	6.408	63	20	30	11	1.315
7	50	50	15	1.901	26	41	46	18	10.526	45	50	30	21	10.272	64	15	5	28	38.636
8	55	45	16	2.657	27	55	34	17	0.011	46	51	42	27	4.469	65	50	70	9	3.808
9	26	59	29	44.373	28	35	16	29	9.220	47	50	15	19	23.388	66	57	72	37	39.609
10	40	66	26	10.999	29	52	26	13	10.994	48	48	21	20	15.534	67	45	42	30	3.377
11	55	65	37	2.687	30	43	26	22	47.673	49	12	38	5	0.547	68	38	33	10	1.660
12	35	51	16	19.683	31	31	76	25	5.812	50	15	56	22	13.179	69	50	4	8	0.096
13	62	35	12	5.438	32	22	53	28	29.811	51	29	39	12	13.596	70	66	8	11	9.754
14	62	57	31	1.039	33	26	29	27	66.052	52	54	38	19	1.771	71	59	5	3	0.504
15	62	24	8	0.544	34	50	40	19	31.535	53	55	57	22	37.227	72	35	60	1	0.058
16	21	36	19	15.441	35	55	50	10	2.741	54	67	41	16	0.663	73	27	24	6	1.646
17	33	44	20	0.310	36	54	10	12	2.515	55	10	70	7	3.899	74	40	20	10	0.003
18	9	56	13	18.418	37	60	15	14	16.553	56	6	25	26	34.514	75	40	37	20	0.228
19	62	48	15	23.978	38	47	66	24	55.485	57	65	27	14	18.681					

Table 4 : SC-W results on 75 customer test problem.

Route #	Mean	Variance	Pr[d > Q]	E[cost]	Distance	Customer number Route sequence
1	53	5.63	0.000	28.91	28.91	17, 40
2	125	54.47	0.000	41.01	41.01	26, 7, 8, 46, 34, 67
3	139	127.73	0.032	139.20	137.56	49, 24, 50, 18, 55, 25, 31, 10
4	142	115.65	0.047	129.18	126.83	61, 22, 64, 42, 41, 43, 56, 23, 63
5	145	109.50	0.076	121.84	118.04	57, 15, 37, 20, 70, 60, 71, 69, 36, 47, 5
6	146	130.98	0.111	97.60	94.41	58, 38, 53, 19, 54, 13, 27, 52
7	148	53.98	0.051	99.23	97.33	11, 65, 66, 59, 14, 35
8	149	147.77	0.183	79.05	74.11	12, 72, 39, 9, 32, 44, 3, 16, 51
9	151	161.19	0.239	60.26	56.85	6, 2, 30, 48, 29, 45, 4
10	166	167.31	0.679	88.63	81.11	33, 73, 1, 62, 28, 21, 74, 68, 75
TOTAL	1364			884.91	856.16	

customer is also the last one on the route, Thus, while 75's inclusion into the route considerably increased the probability of the route failing, this was compensated for by a relatively small penalty in the event of such failure (since it occurred at the last stop which is very close to the depot). In order to measure the quality of the results obtained by the SC-W algorithm, we run on the same problem a version of the Clark and Wright algorithm which is presented in [9] (we call this version of Clark and Wright algorithm the modified C-W) and in which the savings term has the following form :

$$s_{ij} = c_{oi} + c_{oj} - c_{ij} + \lambda P_i + \lambda P_j - \lambda P_{ij}$$

where P_i = probability that the route customer i is on fails

P_j = probability that the route customer j is on fails.

P_{ij} = probability that the combined route fails, and

λ = penalty incurred when a route fails.

The results of this run are presented in Table 5. The routes designed by this procedure only present a viable routing solution for a penalty value of 29 or more. This result confirms the same result noted by Stewart et al. in [10]. The number of routes obtained is 11, regardless of the value λ (for $\lambda > 29$). For those routes, the probability of route failure is quite small in comparison to the SC-W results (.067 in the worst case). When evaluating the expected cost for the resulting system, we present two values : The first one, $E[\text{cost}]$, is the value for the routing system obtained at the end of the modified C-W procedure. The second value for the expected routing cost, $E[\text{cost}]_{\min}$, represents the value after having examined the routing directions of the final solution for all the routes obtained and having picked the best ones. The expected routing

Table 5 : Modified C-W results on 75-customer problem

$\frac{\text{Pr}[d > Q]}{\text{min}}$		λ	Number of routes	Distance	Fullest route		Emptiest route		$E[\text{cost}]$	$E[\text{cost}]_{\text{min}}$
max	Mean				Variance	Mean	Variance	$\text{Pr}[d > Q]$		
1.000	1.000	10.0	1	872.11	1364	1074.19	1.000	--	3431.84	3431.84
1.000	1.000	20.0	1	889.44	1364	1074.19	1.000	--	3234.19	3234.19
1.000	1.000	25.0	1	904.19	1364	1074.19	1.000	--	3442.66	3238.43
1.000	1.000	28.0	1	904.19	1364	1074.19	1.000	--	3442.66	3238.43
1.000	1.000	28.5	1	904.19	1364	1074.19	1.000	--	3442.66	3238.43
0.000	0.067	29.0	11	1016.34	145	99.95	0.067	3.38	1024.85	1019.89
0.000	0.067	30.0	11	1016.34	145	99.95	0.067	3.38	1024.85	1019.89
0.000	0.678	40.0	11	1016.34	145	99.95	0.067	3.38	1024.85	1019.89
0.000	0.031	60.0	11	1031.55	140	114.02	0.031	9.00	1035.21	1032.79
0.000	0.031	80.0	11	1031.55	140	114.02	0.031	9.00	1035.21	1032.79
0.000	0.008	200.0	11	1036.46	134	97.42	0.004	16.38	1037.31	1037.00

cost for the SC-W algorithm is 884.9; this is significantly better than the 1019.9 expected routing cost for this modified C-W. The SC-W also dominates the modified C-W in the length of the designed routes, as well as in the number of routes.

5. CONCLUSION

In this paper, we demonstrate that there are certain cost implications of a stochastic vehicle routing problem which do not occur on a deterministic routing, but which can be accounted for when designing a solution to the problem. As indicated in [9], the greater the complexity of the SVRP considered, the greater the computer-time required for its solution. In our case, the SC-W algorithm required 61 CPU seconds on a CDC CYBER 173 using FTN5 compiler, while one run of the modified C-W took only 12 CPU seconds.

Simple VRP algorithms such as the Clarke-Wright heuristic can be easily modified for adaptation to a stochastic environment, as we have illustrated. When the relevant costs of routing are considered, one should not be deterred by high probabilities for route failures, since by permitting a higher probability of route failure, even taking the cost of such failures into consideration, the ultimate expected routing cost is actually reduced. In our test problem we always used the cost of going twice the distance from the depot to a customer as the penalty for service failure. One might argue that this is too high a penalty. We do not defend the penalty we adopted; we chose it since it represents an upper bound on the actual penalty. The results of our test problem speak for themselves (Table 4).

REFERENCES

- [1] BODIN, L., GOLDEN, B., ASSAD, A. and BALL, M., "The State of the Art in the Routing and Scheduling of Vehicles and Crews", Computer and Operations Research, Special issue, (June 1983).
- [2] CHARNES, A. and COOPER, W., "Chance-constrained programming", Management Science 6 (1959) 73-79.
- [3] CHARNES, A. and COOPER, W., "Deterministic equivalents for optimizing and satisfying under chance constraints", Operations Research 11 (1963) 18-39.
- [4] CHRISTOFIDES, N. and EILON, S., "An algorithm for the vehicle dispatching problem", Operational Research Quarterly 20 (1969) 309-318.
- [5] DROR, M., "The Inventory Routing Problem", Working Paper MS/S, College of Business and Management, University of Maryland at College Park (1983).
- [6] DROR, M. and BALL, M., "Inventory Routing : Reduction from an Annual to Short Period Problem", Publication #353, Centre de recherche sur les transports, Université de Montréal (1984).
- [7] EILON, S., Watson-Gandy, C. and CHRISTOFIDES, N., Distribution Management : Mathematical Modeling and Practical Analysis, Hafner Publishing Co., New York (1971).
- [8] GOLDEN, B. and STEWART, W.R., Jr., "Vehicle routing with probabilistic demands", in : D. Hogben and D. Fife, Eds., Computer Science and Statistics : Tenth Annual Symposium on the Interface, NBC Special Publication 503 (1978) 252-259.
- [9] STEWART, W.R., Jr. and GOLDEN, B.L., "Stochastic vehicle routing : A comprehensive approach", European Journal of Operational Research 14 (1983) 371-385.
- [10] STEWART, W.R., Jr, GOLDEN, B.L. and GHEYSENS, F., "A Survey of Stochastic Vehicle Routing", Working paper MS/S, College of Business and Management, University of Maryland at College Park.